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Calculation of depleted wind resources near wind farms

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Abstract Traditional wind resource maps include wind distribution, energy density and potential power production without wake effects. Adding wake effect to such maps is feasible by means of a new method based on Fourier transformation, and the extra computational work is comparable to that of the basic wind resource map. The method is mainly intended for mapping inter-farm wake effects. It will work for all linear wake models and may even be extended to complex terrain by certain simplifying assumptions. The method is implemented for the Park model and Fuga models. A test example shows that these models predict different wake development on an inter-farm length scale.

Keywords Wind resource, map, wake, Fourier transformation, wind-farm clusters

1 Introduction

Wakes from neighbouring wind farms are of interest for developers of new farms, owners of existing farms and planning authorities. Inter-farm wake effects could reduce the production of existing farms and new wind farms may produce less than expected. Planning authorities will usually keep inter-farm wake-losses at an acceptable level by reserving turbine-free zones around each wind farm. This policy will both reduce actual energy losses and financial costs related to the uncertainty of future productions. Wind farm separation should however not be too large, as this could mean wasting limited areas with favourable conditions for wind energy.

2 Method

Maps of wind resources is useful for wind energy planing, but they usually do not include wake

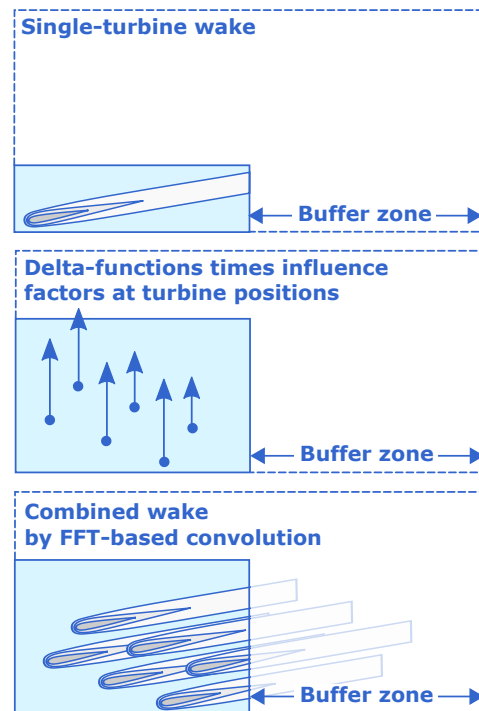


Figure 1: Wake duplication

effects. The aim of this article is to present a new method, which include wake effects in such maps.

2.1 Wake duplication by FFT

The main idea of the new algorithm is to construct maps of wakes from multiple turbines by convolution of the velocity field behind a single turbine and maps of thrust-related influence factors for all turbines. This convolution is done by Fast Fourier Transformation (FFT), which is more efficient than direct computation. The influence factors depend on wind conditions and they are evaluated just like in a traditional annual energy production (AEP) calculation. First we sort turbines after distance along wind direction, then

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we estimate wake effects from upwind turbines exposed to the free wind, and in a progressively manner we calculate sheltered wind speeds, turbine thrust and wakes further downwind in the wind farm. The calculation time for this part of the algorithm is equal to that of standard AEP algorithms. Extra time is needed for convolution of maps of wakes and influence factors, but the Fourier transformations only depends on the number of turbine types, not on the number of turbines. Thus, the method becomes particularly useful for big wind farms.

The algorithm is illustrated by the sketch in Figure 1 and listed as pseudocode in Appendix B. The FFT-based solutions are cyclic, so the convoluted field needs a buffer-zone to avoid wrap-around effects of wakes crossing boundaries of the map part of interest. The necessary width or height of the extended map equals the sum of the widths or heights of the wake- and influence-factor maps. For many wind directions it is possible to trim the wake map and thereby optimize the buffer zones as in Figure 1. The calculations are further optimized by use of the FFTW [1] algorithm.

The method should work for all linear wake models, i.e. models where we can add solutions from several turbines and scale fields of velocity deficits by factors depending on turbine thrust. The method has been implemented for two models - Fuga [2], based on linearized CFD, and the Park [3] model. Park is not linear, since it evaluates the combined wake effect of multiple turbines by the root of the sum of the squared wake contributions. For this particular model, we modify our method by squaring single-wake fields and maps of influence factors before convolution and finally take the square root of the sum of fields for all turbine types. The algorithm parts special for Park are highlighted by blue color in Appendix B.

2.2 Wind resource maps

Maps of wind farm wakes are calculated for every wind speed and direction and then combined with a basic wind resource grid, which in this implementation is imported from WAsP¹. This input provides frequency of occurrence in a number of wind-direction sectors and sector-wise Weibull distributions of the free wind speed for every grid node.

There are two traditional ways to present the wind resources, either by wind energy density or by annual energy production (AEP) of a reference turbine. These calculations are made by

probability-weighted integrals of kinetic energy or turbine production for all grid nodes and wind conditions. Appendix A explains how to integrate over wind speed with and without wake effect. In addition we integrate over wind direction. To ensure statistical representability the default number of sectors in WAsP is limited to twelve, but for wind resources estimates with wake effects we must divide these sectors into sub-sectors with a directional resolution sufficiently fine to catch wakes from all turbines within a reasonable distance. A resolution of three degrees seems adequate for the Park model, but wakes predicted by Fuga are narrower so we select a resolution of one degree. In lack of detailed information we apply the same Weibull distribution for all sub-sectors within each sector and distribute the frequency evenly.

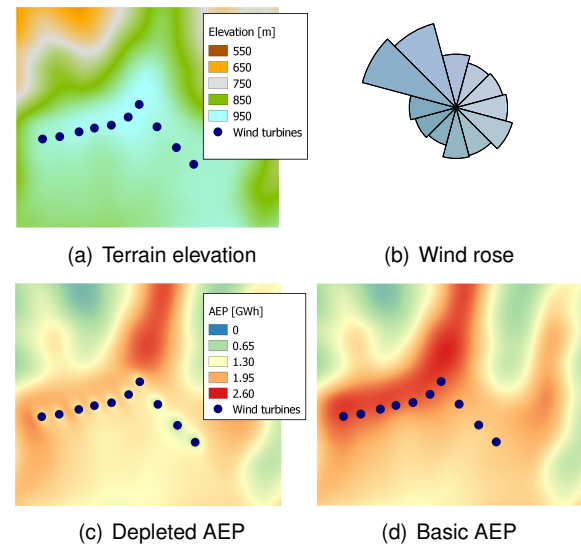


Figure 2: Wind farm in complex terrain - terrain elevation, wind rose plus and resource maps with and without wake effects.

2.3 Complex terrain

Certain simplifying assumptions allow us to apply a linear wake model such as Park for flow in complex terrain. We assume that the wake centreline follows the terrain with a constant distance from the ground, but otherwise the wake geometry is unaffected by terrain topography. A further assumption is that the velocity deficit at the wake-exposed turbine scale by the same speed-up factor as the ambient wind. In the present model we evaluate the thrust of each turbine at wind speeds corrected for speedup relative to a reference site. The wake effects found

¹ <http://www.wasp.dk>

Table 1: Calculation times for resource maps with 102×35 grid nodes. The number of turbines are shown in parenthesis after the wind farm name.

	Basic	Depleted	
	WAsP	FFT	Direct
Nysted (72)	7.8 s	9.2 s	228 s
Rødsand (90)		9.6 s	286 s
Both (162)		14.0/10.8 ^a s	515 s

^aThe shorter time is for one turbine type only.

by FFT-based convolution applies to wind speeds at the reference site, which may differ from local wind speeds. When predicting the wake effect at a new site we therefore first find the corresponding wind speed at the reference site, then interpolate in solutions for a range of reference winds, and finally correct that wind by the speedup at the site of prediction. In the context of resource modelling this does not increase the work load significantly, since we already have to evaluate wake effects for multiple wind speeds. We just have to ensure that the range of reference-site wind speeds is wide enough to enable wake estimates for the range of turbine operation at all positions in the map.

Figure 2 shows results from a site in northern Portugal near $40^\circ 52' 30''$ N $8^\circ 5' 34''$ W within complex terrain. The displayed wind distribution is based on measurements [4], the terrain elevation map is downloaded from SRTM² data set, and the chosen turbine type is a Gamesa 58. The wind-farm layout is entirely fictional and the turbines are probably deployed with too little separation. The only purpose is to demonstrate the method.

The wind resource is displayed as AEP for a reference turbine of the same type as the turbines in the wind farm, and the two maps are calculated with and without wake effects. As expected, the largest AEP reduction is observed near each turbine and there seem to be a directional dependence corresponding to the wind rose.

2.4 Computational efficiency

Table 1 displays calculation times for resource maps with 102×35 grid nodes like the displayed map in Figure 3. The maps cover the area around Nysted and Rødsand wind farms containing 72 and 90 turbines, respectively. The

tests were made on a Windows PC with an i7-2760QM processor and parallel processing was not attempted.

The basic resource map calculated by WAsP contains no wake effects. The depleted wind resource maps are based on local wind climates and they are both calculated by the new FFT-based method and direct computation. The FFT-based calculation for the combined wind-farm cluster is repeated for a shared turbine type and two distinct types. Adding wake effects to the basic wind resource map increases the calculation time by a factor of 2.2–2.8 for the chosen examples and present implementation. For direct computation this factor increases to 30–67. The calculation for both wind-farm cluster spends 4.0 s (29%) on Fourier transformations within the FFTW library, 1.4 s (10%) preparing depleted resource maps by the wake maps, see Appendix A, and 2.3 s (16%) finding influence factors. The latter task might be skipped if the routine was integrated into a resource assessment program like WAsP, since presumably these results are available as part of the wind-farm production estimates.

3 Example

Figure 3 shows wind resource depletion in the vicinity of the Rødsand wind farm, with focus on the neighbouring Nysted wind farm situated 2.5 km to the East. The wind data are taken from another site in Denmark, so results are not entirely realistic and therefore only shown as a relative reduction. The purpose is mainly to demonstrate the methods and to compare the Fuga and Park wake models for inter-farm wake effects. The map is relatively coarse, so details inside the Rødsand wind farm are not shown.

Fuga is developed for offshore wind energy and assumes flat terrain and uniform surface roughness. The solution for the three-dimensional wake embedded in the atmospheric boundary layer is based on linearized flow equations. Surface roughness and atmospheric stability are input parameters which determine the ambient wind profile and eddy diffusivity.

The Park model does not make any assumptions on the terrain, and the expanding wake is simplified to a cone parameterized by a wake decay factor defined as the tangent of half the cone opening angle. Wake expansion depends on turbulence, so it is recommended to select a smaller wake decay factor for offshore projects than for onshore projects. Independent teams in the Offshore CREYAP part one benchmark used

²<http://www2.jpl.nasa.gov/srtm>

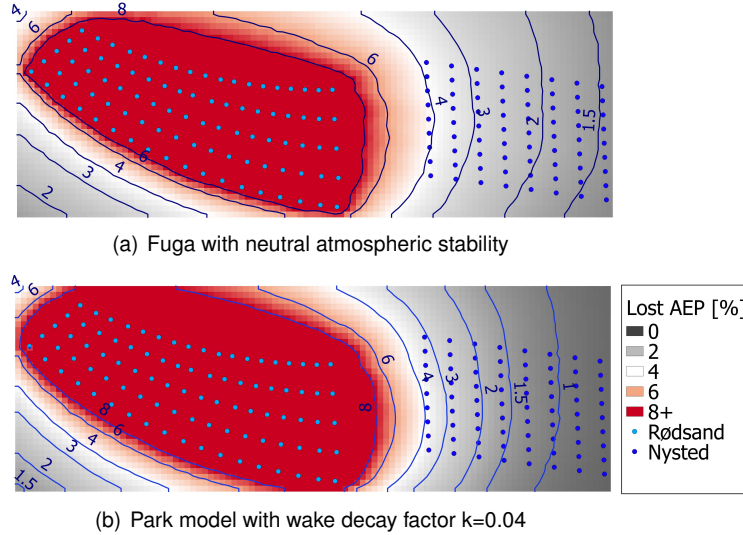


Figure 3: Lost AEP [%] near Rødsand wind farm.

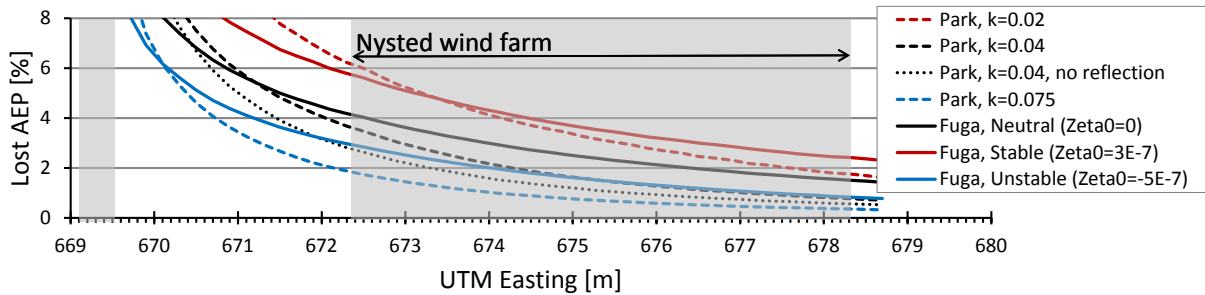


Figure 4: Predictions of lost AEP [%] along an E-W transect through the centre of Nysted Wind Farm using the Park and Fuga wake models with variable model parameters.

Park wake-decay factors in the range $k=0.03$ - 0.075 when modelling the Gwynt y Môr offshore wind farm [5], so there is little consensus on the appropriate value of this parameter.

The predominantly wind direction is west. This introduces an asymmetry in the wind resource depletion, which generally is more significant in easterly directions. The two solutions, Fuga for neutral stability and Park with $k=0.04$, predicts similar reductions ($\Delta AEP=4\%$) for the closest turbines in the Nysted wind farm. The predicted wake effect of Park decays faster with distance than that of Fuga. Figure 4 shows the same tendency for other wake model parameters. It is not possible to match Park and Fuga results for all distances.

4 Discussion

It is hoped that the suggested method will be useful in the early stages of wind farm planning where the layout of a new wind farm has not been

planned in detail. The depleted wind resource map could assist decision makers when determining the appropriate wind farm separation. It could also provide an initial estimate of financial cost related to inter-farm wake effects.

Depleted wind resource maps might also be used for generating an initial turbine layout for optimization of wind farms in complex terrain. An idea could be to place each turbine at the position with best production potential in wind resource map including wakes from previously located turbines. The resulting layout would probably need further optimization by an advanced algorithm.

A problem related to applications in complex terrain is that the method ignores horizontal streamline deflection and local variation of the wind direction. It is also debatable whether the development of the wake dimensions should be independent on terrain topography. However, this critique could also be raised against existing wake models such as the Park in WASP.

5 Conclusions

The suggested FFT-based method for depleted wind resource maps is much faster than direct calculation. The calculation time will depend on the map size, the number of turbine sites and the number of distinct turbine types. The presented example suggests calculation times almost comparable to that of a basic wind resource map. This should make the method feasible for practical applications.

The two offshore wind farms Nysted and Rødsand were used as a test case. Calculations were repeated with the Fuga and Park wake models and seemed relatively sensitive to model parameters. Choosing an appropriate atmospheric stability for Fuga or wake decay parameter for Park is an important but non-trivial task. Unfortunately, it does not seem possible to select a universal wake decay parameter which will match Park with Fuga results for all distances.

Acknowledgements

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- [6] I. Troen and E. L. Petersen. *European Wind Atlas*. Risø National Laboratory, Roskilde, DK, 1989.

Appendix A

Following the European Wind Atlas [6] we approximate the wind distribution by a Weibull distribution

$$p(u) = \frac{u}{k} \left(\frac{u}{A} \right)^{k-1} \exp \left[- \left(\frac{u}{A} \right)^k \right] \quad (1)$$

where A and k are Weibull scale and shape parameters. The turbine power curve is a function of wind speed and usually available as a piecewise linear function.

$$P(u) = \frac{P_i(u - u_{i-1}) + P_{i-1}(u - u_i)}{u_i - u_{i-1}} \quad \text{for } u_i \leq u < u_{i+1} \quad (2)$$

The accumulated production for a given direction is found by a probability integral resulting in the following sum.

$$P = \sum_{i=1}^{N-1} \Delta P_i \quad (3)$$

where N is the number of reference points in the power curve and

$$\Delta P_i = \begin{cases} (P_i - P_{i-1}) \frac{G_k(\alpha_i) - G_k(\alpha_{i-1})}{\alpha_i - \alpha_{i-1}} & \text{if } \alpha_i \neq \alpha_{i-1} \\ (P_i - P_{i-1}) \exp(-\alpha_i^k) & \text{if } \alpha_i = \alpha_{i-1} \end{cases}$$

Here $\alpha_i = u_i/A$ is a dimensionless speed and $G_k(\alpha)$ is short-hand notation for a function involving the incomplete gamma function.

$$G_k(\alpha) = 1/k \cdot \Gamma(1/k, \alpha^k)$$

The power curve is either used directly or interpolated for wake-corrected wind speeds depending on whether we estimate energy production with or without wake effects.

The average wind speed and the energy density are found by analytical integration of the Weibull distribution

$$\begin{aligned} \langle u_{\text{free}} \rangle &= A \Gamma(1 + 1/k) \\ E_{\text{free}} &= 1/2 \rho A^3 \Gamma(1 + 3/k) \end{aligned} \quad (4)$$

For the depleted wind resource map we assume a linear variation of the wake-corrected wind speed v as function of the free wind speed

u . The variation between two reference points is expressed as

$$v = \frac{v_i(u - u_{i-1}) + v_{i-1}(u - u_i)}{u_i - u_{i-1}} \quad (5)$$

By analogy with equation 2 we can calculate average sheltered wind speed by equation 3 when substituting power P_i by sheltered wind speed v_i . For calculation of energy density with wake effects we expand the cube of the sheltered wind speeds as

$$v^3 = c_1^3 u^3 + 3c_1^2 c_2 u^2 + 3c_1 c_2^2 u + c_2^3 \quad (6)$$

using the abbreviations

$$\begin{aligned} c_1 &= \frac{v_i - v_{i-1}}{u_i - u_{i-1}} \\ c_2 &= \frac{v_i u_{i-1} - v_{i-1} u_i}{u_i - u_{i-1}} \end{aligned}$$

The probability-weighted integral of the cube of the sheltered wind speed v^3 between to reference wind speeds u_{i-1} and u_i is calculated by

$$\begin{aligned} I_i &= \int_{u_{i-1}}^{u_i} v^3 p(u) du \\ &= c_1^3 [M_3(u_i) - M_3(u_{i-1})] \\ &\quad + 3c_1^2 c_2 [M_2(u_i) - M_2(u_{i-1})] \\ &\quad + 3c_1 c_2^2 [M_1(u_i) - M_1(u_{i-1})] \\ &\quad + c_2^3 [M_0(u_i) - M_0(u_{i-1})] \end{aligned} \quad (7)$$

with

$$M_n(u) = A^n \Gamma(1 + n/k, (u/A)^k)$$

By this result we calculate the energy density with wake effect as

$$E_{\text{wake}} = \frac{1}{2} \rho \sum_{i=2}^N I_i \quad (8)$$

The results shown in this appendix are for a fixed wind direction only. For actual wind resource estimates we will have to do integrate over wind direction.

Appendix B

```
1: for each wind direction d do
2:   optimize map buffer zone
3:   for each turbine type t do
4:     find singleWakeMap for direction d
5:     singleWakeMap  $\leftarrow$  square(singleWakeMap) ▷ PARK model only
6:     fftSingleWakeMap  $\leftarrow$  FFT(singleWakeMap)
7:   end for
8:   Sort turbines after distance along wind direction
9:   for each turbine site i do
10:    lookup speedup at Site[i] relative to reference site
11:    for each upwind turbine site j do ▷ Loop over sites upwind of site[i] only
12:      find position relative to Site[i]
13:      for each reference wind speed k do
14:        freeSpeed[j]  $\leftarrow$  speedup[j]·refSpeed[k]
15:        lookup reducedSpeedTable[j,k] ▷ This table is updated in line 24
16:        lookup thrust-coefficient for turbine type of upwind Site[j]
17:        find wakeContrib of Site[j] at Site[i] and correct for speedup
18:        wakeContrib  $\leftarrow$  square(wakeContrib) ▷ PARK model only
19:        add wakeContrib to combiWake[k]
20:      end for
21:    end for
22:    for each local wind speed k do
23:      freeSpeedTable[i,k]  $\leftarrow$  speedUp[i]·refSpeed[k]
24:      reducedSpeedTable[i,k]  $\leftarrow$  speedUp[i]·(refSpeed[k]-combiWake[k])
25:    end for
26:  end for
27:  for each wind speed k do
28:    clear wakeMap[k,d]
29:    for each turbine type t do
30:      for each turbine site i do
31:        if Site[i] has turbineType[t] then
32:          lookup reducedSpeed in freeSpeedTable[j,k]
33:          lookup thrust for Site[k] and find influenceFactor
34:          influenceFactor  $\leftarrow$  square(influenceFactor) ▷ PARK model only
35:          add influenceFactor at position of Site[i] to influenceFactorMap
36:        end if
37:      end for
38:      fftInfluenceMap  $\leftarrow$  FFT(influenceFactorMap)
39:      combiWakeMap[t]  $\leftarrow$  inverseFFT(fftSingleWakeMap[t]·fftInfluenceMap)
40:      combiWakeMap[t]  $\leftarrow$  sqrt(combiWakeMap[t]) ▷ PARK model only
41:      add combiWakeMap[t] to wakeMap[k,d]
42:    end for
43:  end for
44: end for
```
